11 Writing in Mathematics: The Role of Intermediate Texts

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A lot of pupils learning in French "Cycle 3" (ages 8-10) are unable to solve meth problems which should normally be easily solved in "Cycle 2" (ages 6-8), because they do not understand the word problems. This study focuses on competencies in addition problem solving by about 600 pupils and as many teachers. It aims to compare results when the phrasing and form of the word problem is varied. Certain tools, based in mathematical literacy, could likely improve the results of pupils by contributing to their understanding of the problem and by the way they operationalize problem solving. This appears to be the case for mediating "graphic figures" like tables, graphic representations, or the way colors can symbolize meaning.

De nombreux élèves connaissent des échecs importants en résolution de problème en raison de difficultés de compréhension des énoncés. Cette étude porte sur la capacité en résolution de problèmes additifs à transformation, réalisée sur environ 600 élèves de cycle 3 de l'école primaire (enfants de 8 à 10 ans) et autant d'enseignants. Elle vise à comparer les résultats en fonction de la forme donnée à l'énoncé. Certains outils, s'inscrivant dans la littératie mathématique, conçus à dessein, pourraient vraisemblablement modifier les résultats des élèves par leur contribution à la compréhension de l'énoncé et par leur opérationnalité résolutoire. Il semble en être ainsi de « figures graphiques » intermédiaires comme une représentation tabulaire, une représentation graphique ou la symbolique des couleurs.

While the whole point of problems is to set mathematical situations which pupils can resolve, they often reveal difficulties of understanding which are located at the level of the language which intervenes as an obstacle in the solving of the problem set. It is common practice to attempt to deal with these difficulties uniquely in the domain of mathematics (working on the meaning of the operations or accumulating solutions to problems) or, independently, in that of language (understanding the wording of the problem). Neither of these two

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approaches to post-learning remedial training are very effective.

If we consider mathematical texts as within the scope of literacy, then writing which, according to Goody (Olson, 2006, p. 85) modifies cognitive processes, could be invoked to understand the wordings of problems. Indeed, mathematics, essentially a science of the written, is sensitive to the effects of writing on reasoning.

This leads to distinguishing two levels of literacy. The first level sets the wording of the problem as a mathematical *text* in the field of the teaching of the French language and focuses on the "processus d'enseignement-apprentissage du français, à partir de données issues de disciplines différentes" ["processes of the teaching-learning of French on the basis of data from different subjects"] (Barré-De Miniac, 2003, p. 7). The wording of problems does indeed come within the scope of the literacy acquired at school which "se réfère à la compréhension et à la production des textes écrits utilisés à l'école" ["refers to the understanding and production of written texts used in school"] (Grossman, 1999, p. 140-141), and is usually defined as the positive aspect of "illiteracy." Sanctioned by these educational issues, literacy enables us to "considérer les compétences des sujets non comme des manques et dans l'absolu, mais en situation et de façon dynamique" ["consider the skills of the subjects not as shortcomings and in the absolute, but in a context and in dynamic fashion"] (Cogis, 2003, p. 103). It is therefore not a question of finding a remedy for the reading difficulties encountered by pupils but of considering, prior to this operation, schemes which would enable pupils to deal with reading the wording of problems (Camenisch et Petit, 2005).

The second level of literacy endorses Olson's remarks (2006, p. 85) stating that writing transforms language into an object of thought. In these instances, the effort enabling an understanding of the wording of problem and facilitating problem solving then involves "intermediate texts" which are considered as reflexive practices which enable us to think, learn and construct (Chabanne et Bucheton, 2000, p. 24). These writings are not restricted to texts but are based on all the possibilities offered by the spatialization of language which includes "des figures graphiques, emblématiques d'usages que seul l'accès à l'écrit rend possibles" ["visual figures, symbolic of uses which are only enabled by access to the written word"] (Lahanier-Reuter), 2006, p. 174).

This chapter is written in this second perspective. A quantitative study carried out in cycle 3 classes (P3 to P5: pupils aged 8 to 10 years), but also among school teachers, enables the formulation of hypotheses on the effect of certain "visual figures" in solving additive problems with one transformation from an initial state to a final state involving an addition or a subtraction. This study pays special attention to the evolution of the performance of the pupils as a function of these figures. How can we take into account the conversion of semiotic registers, articulating texts, tables, graphs, etc. in the context of solving additive problems with one transformation? What are the texts which help the pupils to enter into the reading/understanding of these statements of the problem?

This study leads us to propose a number of graphic tools, tested in an exploratory study in a Cycle 3 class (P3 to P5), which could be taught as from Cycle 2 (P1 and P2: pupils aged 6 and 7 years) to improve the results in solving of certain types of problems.

1. Test: Use of "Visual Figures" in Problem Solving

A quantitative study carried out with over two hundred pupils for each of the three levels in Cycle 3 concerned the solving of additive problems with one transformation, the structures of which were studied by Gérard Vergnaud (1986). These tests were carried out in the first semester of the school year 2011-2012 in classes in several different "*académies*" (National Education administrative regions). The statement of the problems is given in the Appendix in their different forms.

Table 11.1: The content of the problems used in the tests

	Story 1	**Story 2**	**Story 3**
Stories	At departure from Paris, there are 547 passengers in the train Paris/Le Mans/Rennes. 324 passengers get off at Le Mans. 223 passengers continue to Rennes.	713 passengers get on the train to Paris/Aix-en-Provence/Marseille. 179 passengers get off at Aix-en-Provence; nobody gets on at this stop. The train transports 534 passengers between Aix- en-Provence and Marseille.	242 passengers get on at Mulhouse in the train to Mulhouse/Strasbourg/Paris. 314 get on the train in Strasbourg where nobody gets off. 556 passengers arrive in Paris.
Statement of the problems	Problems 1 a-1; c-1; b-1	Problems 2 b-2; a-2; c-2	Problems 3 c-3; b-3; a-3

1.1 Arrangements set up in Cycle 3 classes (P3 to P5)

To evaluate the impact of the "material" presentation of the wording of the question on the pupils' results, we proposed three additive problems with one transformation in three different forms. The three problems proposed were based on what we will refer to as "stories," that is, the explicit wording of all the facts in chronological order, with no additional information (Camenisch and Petit, 2008).With the exception of a few minor variations, these three stories have the same in-depth structure established according to the marking defined by Gérard Vergnaud (1986).

Table 11.2: In depth structure of the stories

Departure	**Stop**	**Arrival**
x passengers	Boarding or alighting of *y* passagers	*z* passagers
Initial state *Ei*	Transformation *T*	Final state *Ef*

The complexity of these wordings can be evaluated in function of the congruence (Duval, 1995, 49) analyses in the following table:

Table 11.3: Congruence of the wording of the problem

	Problem 1	**Problem 2**	**Problem 3**
Wording	Ei T (Ef)	T Ef (Ei)	Ei Ef (T)
Order of wording	chronological	non chronological	non chronological
Operation to be performed	subtraction	addition	subtraction
Verb used	get off	get off	get on
Data processing	order of wording 547—324	order indifferent 179 + 534 ou 534 + 179	reverse order from the order of the wording 556—242
Congruence	Congruent operation	Operation not congruent with the verb	Operation not congruent with the verb and the order of the wording
Type of wording	congruent	non congruent	non congruent

Note: The order of the three periods follows Vergnaud's classification: Ei for "Etat initial" (Initial State), T for transformation, Ef for "Etat final" (Final State). The brackets indicate the value sought.

Given their non-congruence, problems 2 and 3 are therefore considered as being more "complex" than problem 1. (A problem is described as "complex" when solving it is not a question of using new skills but of organizing the skills acquired in language or well mastered mathematical tools, such as addition and subtraction.) Each of these problems originates in one of the stories (1, 2 and 3) and has been given in three different forms: a, b and c.

Table 11.4: Form of the wording of the problem

Forms	**a**	**b**	**c**
Presentation	**Text(Txt)**	**Tableau (Tb)**	**Graphique (G)**
Order of the information	Order of the statement	Chronological order is implicit (reading direction)	Chronological order is explicit (axis)
Values	Numerical	Numerical	Numerical and in a graph *

** To facilitate carrying forward, the values have been rounded in the graph.*

1.2 Provisions for Taking the Tests and Correct Results

The tests were administered on three different days in the same week. In order to avoid any communication of the answers, each day one-third of the pupils answered Document A, one-third Document B and one-third Document C (appendix 1). The general principle was to consider an answer was correct when a math operation which gave the solution was correctly written, which refers to the meaning of the operation. A gap-fill addition could replace a subtraction.

For the wording given in c, in c-1 and c-2, the correct answer was the correct insertion in the histogram of the number of passengers. In c-3, the correct result, or the correct writing of an identity which gives the solution were considered to be correct. Our choice avoided errors in calculation (arithmetic) introducing a bias into the results, thus giving a better idea of the way in which the pupil understands the situation described.

Table 11.5: Results of the right answers (in percentage)

	Problem 1 (Pb1)			**Problem 2 (Pb2)**			**Problem 3 (Pb3)**		
	Txt	Tab	G	Txt	Tab	G	Txt	Tab	G
	a-1	b-1	c-1	a-2	b-2	c-2	a-3	b-3	c-3
CE2 (P3) /207*	86	57	25	47	44	21	29	32	20
CM1 (P4)/220	89	72	44	49	58	35	52	43	36
CM2 (P5)/230	97	89	74	56	74	65	71	65	70

** Grade /numbers of students*

It is not our intention to show that pupils have more difficulty in resolving non-congruent problems (Pb2 and Pb3) than congruent problems (Pb1), even if these findings do confirm Duval's research (1995). The test performances indicate another major trend: the scores obtained rise in each of the columns between CE2 (P3) and CM2 (P5), that is, no matter in what form the questions are presented. However, this difference in variation is dependent on the level of the pupils and the form of the questions.

1.3 Analysis of the Findings by Level

In CE2 (P3) (Appendix 2) Pb1 is solved more successfully that Pb2, which is solved more successfully thanPb3; this is the case no matter what the form of the problem a, b or c. (For the details of the scores see Appendices 2, 3, and 4.). For Pb2 and Pb3, there is thus no significant difference between a question posed in table form and a question posed in written form, but there is a significant difference between a question given in written form and one given in a graphic shape, with preference for the written form. At this level, visual figures do not therefore appear to make a distinct improvement in scores.

In CM1 (P4) (appendix 3), the first observation for CE2 (P3) is no longer true, only Pb1 stands out from the other two, no matter what the form of presentation of the problem. Problems 2 and 3 are not differentiated in terms of scores for written texts (a-2: 49%; a-3: 52%) and visual figures (c-2: 35%. c-3: 36%). However, there are significant differences in scores between written forms and tables, once with preference for the written form, (a-3: 52%; b-3; 43%) and once with preference for the table (a-2; 49%: b-2; 58%). Therefore, at this level, a difficult problem given as a table can give significant improvement in scores when compared with the written form.

In CM2 (P5) (appendix 3) the first observation made for CE2 (P3) is no longer true. While problem 1 still obtains much better results than the other two, independent of the form of presentation of the questions, Pb3 seems overall to be resolved better that Pb2 in written and graphic shape [(a-2: 56%; a-3: 71%) (c-2: 65%, c-3: 70%)]. Tables give significantly better scores than the written form (in b-2), confirming the scores for CM1 (P4) and, for the first time, the visual shape gives significantly better scores for Pb2 (c-2: 65%) than those obtained by the problem stated in writing (a-2: 56%).

To conclude the analysis of pupils' scores for "visual figures," it appears that the hypothesis can be made that, to solve a complex problem, tables and shapes are modes of presentation of the questions which can influence pupils' performance. This could lead us to suggest their use as intermediate texts between questions posed in written form and the search for an answer. It is also appropriate to evaluate the way in which the performances of pupils develop in the course of Cycle 3 depending on the form in which the questions are posed.

1.4 Evolution of Performances as a Function of Tables and Visual Figures.

The table below shows changes during Cycle 3 (P3 to P5) of the rate of success of pupils as a function both of the level of teaching and of the visual figure in the text. It is important to note that this is not an example of one and the same cohort of pupils observed at different levels.

Table 11.6: Evolution of pupils' success rates (in percentage)

		Txt a-	**Tab b-**	**Graph c-**
Problem 1	**CM1/CE2 (P4/P3)**	1,03	1,26	1,76
	CM2/CM1 (P4/P5)	1,09	1,24	1,68
	CM2/CE2 (P5/P3)	1,13	1,56	2,96
	Evolution in %	13	56	196
Problem 2	**CM1/CE2 (P4/P3)**	1,04	1,32	1,67
	CM2/CM1 (P4/P5)	1,14	1,28	1,86
	CM2/CE2 (P5/P3)	1,19	1,68	3,10
	Evolution in %	19	68	210
Problem 3	**CM1/CE2 (P4/P3)**	1,79	1,34	1,80
	CM2/CM1 (P4/P5)	1,37	1,51	1,94
	CM2/CE2 (P5/P3)	2,45	2,03	3,50
	Evolution in %	145	103	250

This table demonstrates that mastering two non-written modes of representation (b and c) rises significantly for tabular mode (rise between 56% and 103%) and in a more surprising manner for visual representation mode (from 196% to 250%). This rise is all the more spectacular in that this mode of representation had never been taught as such to the pupils. This demonstrates in particular the accessibility of these tools to pupils.

However, one objection persists. One might think that changes in pupils' performances were the direct consequence of their psycho-genetic development and that it is therefore not necessary to develop the teaching of tools of this sort. We consider it is possible to refute this point of view by the scores which follow.

1.5 Arrangements at Teacher Level

A problem set by G. Vergnaud was given to 648 teachers in Cycles 2 and 3. This was a random sample of teachers taken from the participants in in-service training courses (lectures) in mathematics in 2012 and 2013 in different regions in France.

> Mr. Durand wishes to renew the electricity in 3 rooms in his house. He estimates that he requires 130 meters of electrical wire, 4 switches, and 9 electric sockets as well as lamp holders. From a previous installation he still has 37 meters of electrical wire that he is going to use. He will therefore have to buy more wire. After finishing his installation, he notices that he has used 4 meters less wire than expected and that he still has 11 meters. How many meters did he buy?

This complex statement of the problem presented lexical difficulties (*estimate* and *expected*) as well as difficulties related to the non-congruent treatment of the data. The table below gives the teachers' answers. The correct answer (100 meters) was only given by 24% of the teachers.

Table 11.7: Results of the teachers' test

Answers	**93 m**	**100 m**	**No answer**	**Others**	**Total**
Total number	387	153	21	87	648
Percentages	60%	24%	3%	13%	100%

The "other" answers varied widely covering a range of twenty-eight different answers between 12 meters and 178 meters with several peaks at 78 meters (21), 104 meters (14), 82 meters (8) and 137 meters (7).

The results, coming from teachers in charge of implementing the learning of solving similar types of problem, refute the hypothesis that psycho-genetic development alone could explain the improvement of the pupils' performances in the course of their education. If this was indeed the case, with a few exceptions, all the teachers would have found the correct answer. Only a very few spontaneously used an intermediary tool to solve the problem.

It is therefore permissible to think that working explicitly on a number of "visual figures" which are considered to be intermediate texts could lead to an improvement in performance in problem solving in the classroom.

2. Intermediate Texts

The two series of tests carried out demonstrate that success is due more to understanding the text of the problem and its functioning, therefore to reading skills, than to the mastery of purely mathematical skills (acquired skills in calculating, like addition and subtraction). It is the understanding of the text, and therefore of the situation, which conditions the choice of operation to be implemented. Solving an addition with a transformation also consists of discovering what is implicit (the command part of the statement) in a text which is cognitively complete (the informative part of the statement). The story behind the statement of the problem is the most explicit cognitive form of the data set. Solving an additive problem with one transformation therefore involves reconstituting the whole story on the basis of a statement containing an implicit element.

2.1 Conversion of Semiotic Registers

We postulate, as does Duval (1995, p 75), that it is appropriate to provide pupils (possibly also their teachers) with different semiotic registers of data representation. These registers then act as intermediate tools, aids to problem solving, required by some pupils or teachers, in order to understand the wording itself and to acquire the degree of operational competence required to solve the problem. They then play a cognitive and meta-cognitive role.

This postulate implies a choice of semiotic registers to be used which differs from the usual iconographical representations in school books, these being of little use in problem solving. The understanding of the wording of problems in written form will then be the outcome of various articulations effected between these registers, which imposes a task of "conversion" of the registers (Duval, 1995, p. 40). It then becomes essential to identify the "meaningful entities" (Duval, 1995, p. 40) in a register, that is, those which enable the problem to be solved, then to make it correspond to meaningful entities in another register. But "la difficulté propre à l'activité de conversion réside essentiellement dans cette discrimination" ["the difficulty inherent in the activity of conversion resides essentially in this discrimination"] (Duval, 1995, p. 77). Two wordings of problems with the same basic story may present different degrees of difficulty depending on their congruence or non-congruence with the associated identity that gives the solution. The cognitive content of the two statements may be strictly identical, whereas the difficulties in understanding the text may present considerable discrepancies.

It is therefore necessary to distinguish what belongs to purely linguistic processing in the text of the wording of a problem, like the wording of the chronology, and what comes under cognitive processing (Duval, 1995, p 84).

Intermediate "visual figures" therefore aim at introducing in *visual* or explicit manner the temporality of the statement of the problem, in order to concentrate on the purely cognitive aspects; which alone enable the solving of the problem. These tools are part of the context of educational literacy in which the "travail de la pensée consiste concrètement dans la manipulation (réutilisation, transformation, invention) de formes sémiotiques, toutes les formes de symbolisation, mathématique, graphique, etc." ["task of thinking consists materially of the manipulation (re-use, transformation, invention) of semiotic forms, in all the forms of symbolization, mathematical, graphs, etc."] (Chabanne et Bucheton, 2000, p. 24). To encourage a reflexive approach and the construction of new meanings (Faure, 2011, p. 22), we have thus developed tools using resources associated with a spatial or symbolic representation. These new tools (pictures, timelines, graphs, colors) have been designed to enable pupils to restrict the purely discursive effects and to highlight the meaningful entities relevant to solving the problem.

2.2 Tables and Timelines

Analyzed by Goody in 1979 as special "visual procedures" (Lahanier-Reuter, 2006, p. 175), Duval defines tables (2003, p. 7) as being an arrangement in lines and columns which "visually separates" the data by delimitation in boxes, thus presenting the information separately. This separation of the units of the discourse, reinforced by drawing lines, is specific to writing. The tables which we designed in the "b" statements thus return to the spatial arrangement of a table by the separation of boxes delimited by lines, organizing in this way the semiotic units expressed by sentences or expressions. But, in contrast to tables, here the "horizontal" margin induces a chronological order. This sort of tabular representation therefore has a close affinity with the timelines used by pupils in other subjects. It can thus contribute to pupils' thinking and constitute a reading aid by enabling them "de discerner [des] unités de sens dans un texte où plusieurs niveaux d'expression, ou de sens, se trouvent fusionnés" ["to have a clearer picture of the units of meaning in a text in which several levels of expression, or meaning, are merged"] (Duval, 2003, p. 8).

Incidental learning is probably the key to pupils' success in this mode of data representation, independent of teaching specific to mathematics. It is therefore possible to imagine that explicit teaching of the use of this type of graphic tool would distinctly improve the pupils' results, by neutralizing the order of the wording of the problem in the statement given in written form. One difficulty remains. It resides in the absence of visual perception of the data and therefore in the difficulty of "seeing" a variation.

2.3 Idiosyncratic Graph

The idiosyncratic graph, combining text and histogram, of the "c" form wordings, situates the periods on an explicit time axis, and gives a visualization of the data by representing it in graphic shape. The difficulties related to chronology disappear while the variations due to the transformation are visually perceptible. To do this, the reader has to combine the meaning of the variation in time and the meaning of the variation in the data (indicated by a "vertical" scale). This register differs significantly from those proposed by R. Damm which, over- contextualized, mix iconographic representation and semiotic representation (Damm, 1992).

While the two preceding tools were based on the restitution of chronological order, it is therefore important to propose a tool enabling the pupils themselves to restore this order. This is what we propose to do with the use of a color code.

2.4 Symbolism of Colors

In the context of the reading and the production of wording of problems, one of the means of chronologically organizing the periods consists of presenting them *visually* by using a color code. We have chosen the conventional order of the colors of the French flag, well known to all pupils in France, to represent the story in its chronological order. This range of colors also clearly shows the six temporal structures which can follow the statement of a problem when the question, which concerns implicit information, is formulated at the end of the wording. The distribution of the periods from the initial state (Ei noted in blue, or B), the transformation (T noted in white or Bc) and the final state "Ef noted in red, or R) is then represented as follows in the wordings of the problem (Camenisch et Petit, 2008, p. 4):

Table 11.8: Repartition of the periods in the statements of problem

Story	Ei - B	T - Bc	Ef - R
Statement 1	Ei - B	T - Bc	(Ef - R)
Statement 2	Ei - B	Ef - R	(T - Bc)
Statement 3	T - Bc	Ei - B	(Ef - R)
Statement 4	T - Bc	Ef - R	(Ei - B)
Statement 5	Ef - R	T - Bc	(Ei - B)
Statement 6	Ef - R	Ei - B	(T - Bc)

Through the symbolism of colors, these representations promote the conceptualization of the difference between order or wording and chronological order without using discourse. It also enables periods, states or transformations to be identified, *simultaneously* with the reading of the wording. By learning and using this visual procedure, the pupils thus become active readers, one of their tasks being to reconstitute the chronological order of the data.

The various graphic tools specifically created to promote improved understanding and the solving of additive problems with one transformation enable a "spatialization" of the wording and neutralize the superficial effect obtained by permutations in time; preference is given to the selection of the entities which are relevant to solving the problem. When several registers can be mobilized on the basis of the same cognitive activity, it is appropriate, not to work independently in each of these registers, but to work in the inter-register space, being careful to make the links between these registers explicit.

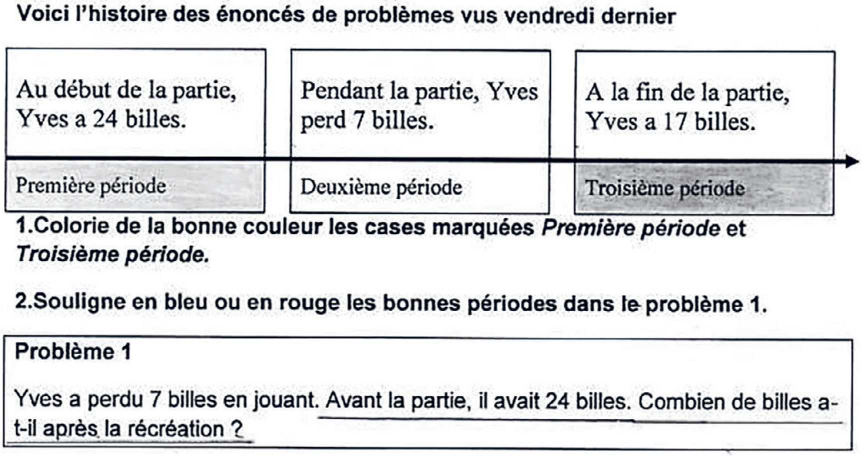
3. Some Tools for Literacy

Consideration of the written texts leads to suggesting activities targeting the explicit learning of the conversion of semiotic registers and the use of these texts in the context of solving additive problems. Tools of this type were introduced in the context of a project on the reading and writing of the wording of additive problems in a CM1 (P4) class in a school in the center of Colmar in January and February 2012. In the framework of this experiment we proposed to support the reading of the wordings of the problems by constructing a reading strategy which explicitly mobilized the conversion of registers of representation. We will now present three tools used in the pupils' productions.

3.1 Story and Problem: A Comparison Between the Two Representations

Pupils were asked to link a series of problems, previously solved, with the under-lying story in which they originated (see Figure 11.1). The story was presented by the teacher in tabular form; each period was specifically named and represented in a box in the table. The chronology was indicated by an arrow. Two working principles implicit in the story were thus revealed: the chronological order and the correlation between period and sentences in the story. The problem was represented in classical written form, with one sentence per period. The wording of the temporal points of reference was scrambled to prevent the pupils from engaging in a selective literal reading to work out the periods.

After identifying the colors in the tables, the pupils had two tasks: to relate the sentence with the statement of the story to the corresponding question in the wording of the problem; and the use of the symbolic color code to recognize the periods and the order of their statement in the written form of the wording.


Voici l'histoire des énoncés de problèmes vus vendredi dernier

Au début de la partie, Yves a 24 billes.	Pendant la partie, Yves perd 7 billes.	A la fin de la partie, Yves a 17 billes.
Première période	Deuxième période	Troisième période

1.Colorie de la bonne couleur les cases marquées *Première période* et *Troisième période.*

2.Souligne en bleu ou en rouge les bonnes périodes dans le problème 1.

Problème 1

Yves a perdu 7 billes en jouant. Avant la partie, il avait 24 billes. Combien de billes a-t-il après la récréation ?

Figure 11.1. Comparison between a story and a statement (pupil's production)

The role of this intermediate text is to differentiate the order of statement of the problem and the chronological order of the story. These texts were to become the support for an explicit reflexive activity to create awareness of this non-congruence factor. In pairs, pupils were then asked to compare, in writing, the wording of the problem and of the story, pointing out the differences between the two texts.

Some pupils focused on the changes in the expression: "The words are not the same in the story and the problem." Others focused on the differences at the level of the sentence or the text: "There is a question in the problem but not in the story and the problem is not in chronological order." "In the problem there is a question and in the story the periods are not mixed."(The spelling of the pupils has been corrected.) All the pupils' replies led to a structuring of the differences between the story and the wording of the problem.

The literacy tools, tables, timelines, and color symbolism enabled this reflexive activity which aimed at showing the characteristics of the problem and its relation with the story behind it.

3.2 Learning Activities and the Conversion of Registers

In the learning activities for the conversion of the registers which we had set up, the pupils were helped to explicitly understand the working of the graph, by filling a blank graph with the data presented in another register of representation. The graph was also a tool for solving the problem, since filling it in was the equivalent of solving the problem.

The first intermediate text presented a problem as a gap-fill story with the three periods given in the boxes, a minimal tabular representation:

Table 11.9. Representation of a problem as a gap-fill story

Monday evening, the temperature is ___ degrees.	During the night of Monday to Tuesday, the temperature drops 5 degrees.	Tuesday morning, the temperature is 7 degrees.

The problem above would correspond in its conventional written form to a statement of the problem similar to problem 2 in the tests [T, Ef, (Ei)], which most of the pupils failed, in which the question, stated at the end relates to the state which corresponded to the first period. The table re-established the chronological order and avoided the interrogative form.

The second intermediate text (Figure 11.2) is an idiosyncratic graph. The task of the pupil consists of localizing the periods of the statement and entering them in the boxes provided, identifying the data of the known states (here the final state alone) and entering them on the graph by coloring, indicating in a sentence the nature of the transformation (here: the temperature falls 5 degrees during the night), examining the transformation to determine whether the temperature was higher or lower on Monday evening and to determine the variation, filling in the graph by entering the value found (here: 12 degrees), checking the coherence of the data entered on the graph and, finally, concluding.

The graph obtained is also an account of the whole story and therefore solves the problem.

The intermediate texts—the gap-fills and the graphs—therefore enable the pupils to carry out conversions of registers of representation and therefore get a better idea of the data of the problem. It is through the increasing use of activities of this type that pupils will learn to understand this system of representation which then appears as a dynamic learning tool.

3.3 From a Story to the Production of the Wording of a Problem

The code of colors becomes a tool for literacy in the context of the production of the wording of a problem in examination conditions on the basis of an imposed or set story and order of wording. Each period of the story is written on a corresponding colored label which the pupils use to produce a statement of the problem respecting the other order of statement which is imposed.

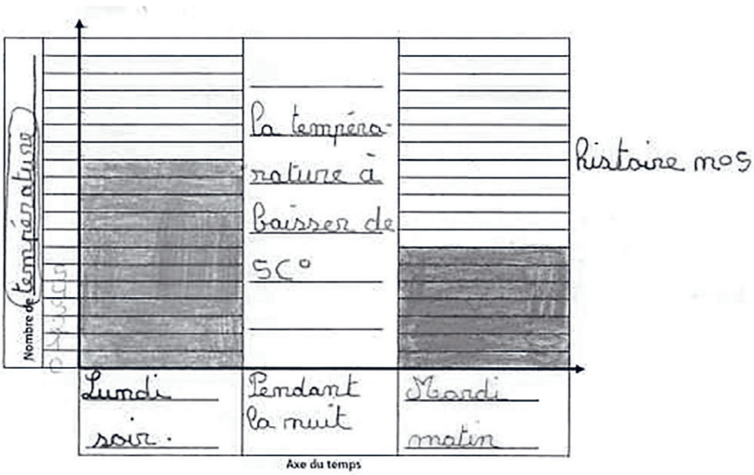
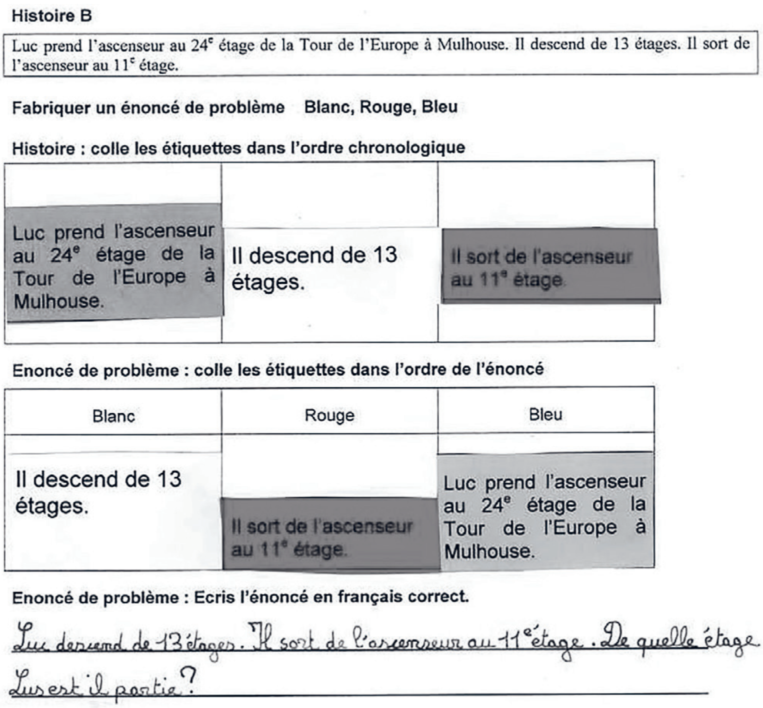


Figure 11.2: Graphic representation of the story (pupils' production)



Histoire B

Luc prend l'ascenseur au 24^{e} étage de la Tour de l'Europe à Mulhouse. Il descend de 13 étages. Il sort de l'ascenseur au 11^{e} étage.

Fabriquer un énoncé de problème Blanc, Rouge, Bleu

Histoire : colle les étiquettes dans l'ordre chronologique

Luc prend l'ascenseur au 24^{e} étage de la Tour de l'Europe à Mulhouse.	Il descend de 13 étages.	Il sort de l'ascenseur au 11^{e} étage

Enoncé de problème : colle les étiquettes dans l'ordre de l'énoncé

Blanc	Rouge	Bleu
Il descend de 13 étages.	Il sort de l'ascenseur au 11^{e} étage.	Luc prend l'ascenseur au 24^{e} étage de la Tour de l'Europe à Mulhouse.

Enoncé de problème : Ecris l'énoncé en français correct.

Luc descend de 13 étages. Il sort de l'ascenseur au 11^{e} étage. De quelle étage Luc est il partie?

Figure 11.3: A written statement of a problem in test conditions (pupil's production). In this production, the pupils were asked to handle blue labels ('Luke takes the lift [. . .]'), white labels ('He goes down [. . .]') and red labels ('He leaves the lift').

This activity also involves all the language difficulties specific to this type of wording, that is the time markers, the substitutes, and the interrogative type (Camenisch et Petit, 2007 & 2008). The aim of this type of activity is ultimately to give the pupils the means of reconstituting the underlying story by the insertion of the reverse operation.

Conclusion

The tests carried out in class and the results of the school teachers reveal the need to introduce pupils and their teachers to strategies which are relevant to the comprehension of wordings of problems. They demonstrate the impact of the use of a visual figure on performances in problem solving. These findings are all the more significant in that no learning of the tools suggested had been set up in class before the passing of the tests. It is therefore permissible to consider that an explicit learning of these tools would enable pupils (and their teachers) to considerably improve their skills in problem-solving. However, these visual figures should not be taught separately because it is through their articulation that each one acquires meaning and efficiency. These figures, instead, enable the construction of meaning by starting with natural language and then leading to improved understanding.

While the use of this type of "visual tool" in remedial classes can improve the performances of pupils in Cycle 3 in solving additive problems, serious thought should be given to considering their learning in Cycle 2, when pupils begin to solve additive problems and to use tables and graphs. Writing in mathematics consists of learning to mobilize various texts, in particular visual and symbolic, to develop the cognitive skills associated with problem solving.

References

Barré-De Miniac, C. (2003). Présentation. *Lidil*, 27, 5-10.

Camenisch, A., & Petit S. (2005). Lire et écrire des énoncés de problème. *Bulletin de l'APMEP*, *456*, 7-20.

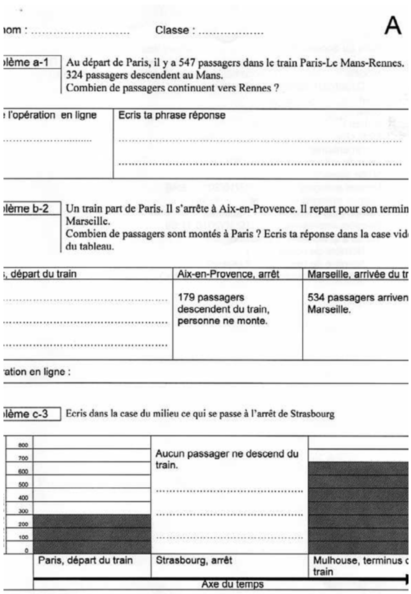
Camenisch, A., & Petit, S. (2007). Projets d'écriture en mathématiques pour mieux comprendre les énoncés de problème. *Actes du 33e Colloque européen de la COPIRELEM.* Dourdan, France: CRDP de Versailles, CDDP de l'Essonne Evry. Cédérom.

Camenisch, A., & Petit, S. (2008). Apprendre à écrire par les mathématiques. In D. Alamargot, J. Bouchand, E. Lambert, V. Millogo, & C. Beaudet (Eds.), *Actes du colloque international De la France au Québec: L'Ecriture dans tous ses* états. Poitiers, France. Available at http://espe.univ-poitiers.fr/apprentissage-et-enseignement-de-la-langue-ecrite-1156331.kjsp?RH=1400759679744

Chabanne, J.-C., & Bucheton, D. (2000). Les écrits intermédiaires. *La lettre de la DFLM*, *26*, 23-27.
Cogis, D. (2003). Marques orthographiques du féminin et pratiques de l'écrit. *Lidil*, *27*, 103-115.
Damm, R. (1992), *Apprentissage des problèmes additifs et compréhension de texte*. Strasbourg: Thèse de Doctorat.
Duval, R. (1995). *Sémiosis et pensée humaine. Registres sémiotiques et apprentissages intellectuels.* Bern: Peter Lang.
Duval, R. (2003). Comment analyser le fonctionnement représentationnel des tableaux et leur diversité? *Spirale*, *32*, 7-31.
Faure, M.-F. (2011). Littératie: Statut et fonctions de l'écrit. *Le Français Aujourd'hui*, *174*, 19-26.
Goody, J. (2006). La littératie, un chantier toujours ouvert. *Pratiques*, *131/132*, 69-75.
Grossmann, F. (1999). Littératie, compréhension et interprétation des textes. *Repères*, *19*, 139-166.
Lahanier-Reuter, D. (2006). Listes et tableaux: Mise en perspective. *Pratiques*, *131/132*, 174-186.
Olson, D. (2006). Littératie, scolarisation et cognition. Quelques implications de l'anthropologie de Jack Goody. *Pratiques*, *131/132*, 83-94.
Vergnaud, G. (1986). Psychologie du développement cognitif et didactique des mathématiques: Un exemple, les structures additives. *Grand N*, *38*, 21-40.

Appendices

Appendix 1



ıom : .... Classe : .... A

ılème a-1 Au départ de Paris, il y a 547 passagers dans le train Paris-Le Mans-Rennes. 324 passagers descendent au Mans. Combien de passagers continuent vers Rennes ?

l'opération en ligne	Ecris ta phrase réponse
....	....

ılème b-2 Un train part de Paris. Il s'arrête à Aix-en-Provence. Il repart pour son termin Marseille. Combien de passagers sont montés à Paris ? Ecris ta réponse dans la case vid du tableau.

, départ du train	Aix-en-Provence, arrêt	Marseille, arrivée du tr
....	179 passagers descendent du train, personne ne monte.	534 passagers arriven Marseille.

ation en ligne :

ılème c-3 Ecris dans la case du milieu ce qui se passe à l'arrêt de Strasbourg

First name: . . . . Class: . . . .

A

Problem a-1: At departure from Paris, there are 547 passengers in the train “Paris-Le Mans-Rennes.” 324 passengers get off at Le mans. How many passengers continue to Rennes?

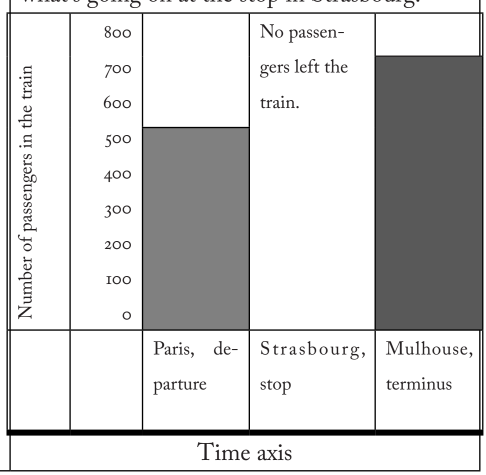
Operation in line	Write the answer

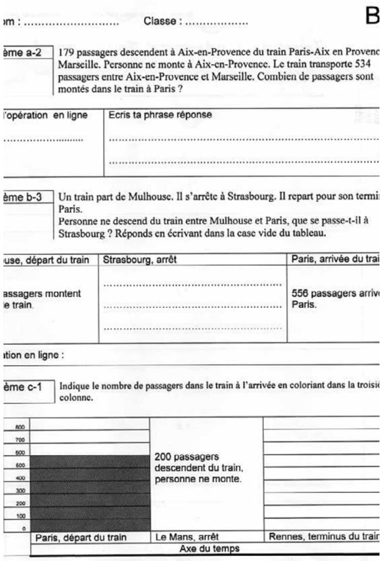
Problem b-2: A train lives Paris. It stops at Aix-en-Provence. It starts again for Marseille, its terminus. How many passengers got on in Paris?

Write the answer in the blank cell of the table.

Paris, departure	Aix, stop	Marseille, terminus
	179 passengers get off the train. Nobody gets on.	534 passengers arrive at Marseille.
Operation in line:		

Problem c-3: Write in the central cell what’s going on at the stop in Strasbourg.





ɔm : .... Classe : .... B

ème a-2 | 179 passagers descendent à Aix-en-Provence du train Paris-Aix en Provenc Marseille. Personne ne monte à Aix-en-Provence. Le train transporte 534 passagers entre Aix-en-Provence et Marseille. Combien de passagers sont montés dans le train à Paris ?

l'opération en ligne	Ecris ta phrase réponse
....	....

ème b-3 | Un train part de Mulhouse. Il s'arrête à Strasbourg. Il repart pour son termi Paris. Personne ne descend du train entre Mulhouse et Paris, que se passe-t-il à Strasbourg ? Réponds en écrivant dans la case vide du tableau.

use, départ du train	Strasbourg, arrêt	Paris, arrivée du trai
assagers montent le train.	....	556 passagers arriv Paris.

ation en ligne :

ème c-1 | Indique le nombre de passagers dans le train à l'arrivée en coloriant dans la troisiè colonne.

First name: . . . . Class: . . . .

B

Problem a-2: 179 passengers get off the train "Paris-Aix-en-Provence-Marseille" in Aix-en-Provence. Nobody gets on the train there. The train carries 534 passengers between Aix-en-Provence and Marseille. How many passengers got on the train in Paris?

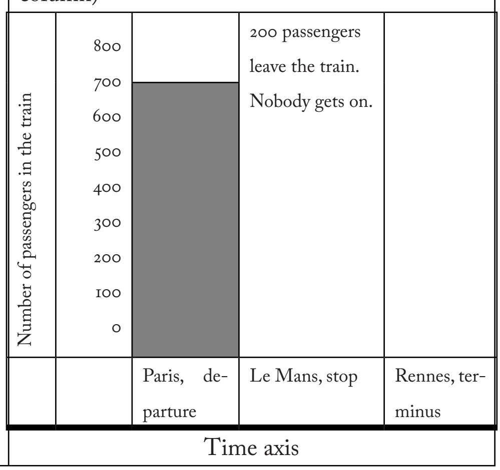
Operation in line	Write the answer

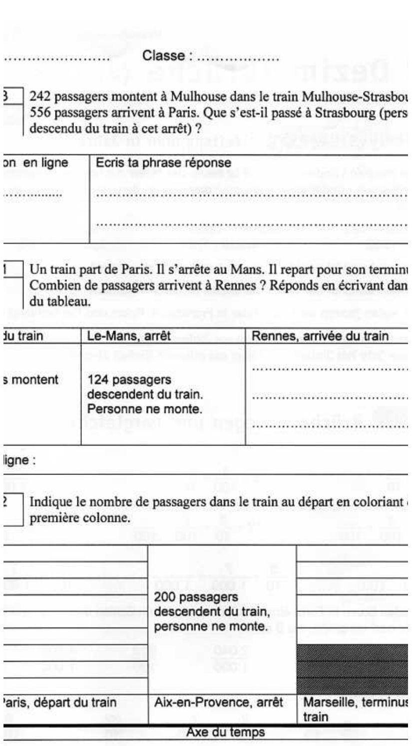
Problem b-3: A train leaves Mulhouse. It stops at Strasbourg. It starts again for Paris, its terminus. Nobody leaves the train between Mulhouse and Paris. What happens in Strasbourg?

Write the answer in the blank cell of the table.

Mulhouse, departure	Strasbourg, stop	Paris, terminus
242 passengers get on the train		556 passengers arrive at Paris.
Operation in line:		

Problem c-1: Color the number of passengers in the train at the terminus (in the third column)





.... Classe : ....

3 242 passagers montent à Mulhouse dans le train Mulhouse-Strasbou
556 passagers arrivent à Paris. Que s'est-il passé à Strasbourg (pers
descendu du train à cet arrêt) ?

on en ligne	Ecris ta phrase réponse
....	....

1 Un train part de Paris. Il s'arrête au Mans. Il repart pour son termin
Combien de passagers arrivent à Rennes ? Réponds en écrivant dan
du tableau.

du train	Le-Mans, arrêt	Rennes, arrivée du train
s montent	124 passagers descendent du train. Personne ne monte.	....

ligne :

2 Indique le nombre de passagers dans le train au départ en coloriant
première colonne.

	200 passagers descendent du train, personne ne monte.	
aris, départ du train	Aix-en-Provence, arrêt	Marseille, terminus train

Axe du temps

First name: .... Class: ....

C

Problem a-3: 242 passengers get on the train "Mulhouse-Strasbourg-Paris" in Mulhouse. 556 passengers arrive at Paris. What happened in Strasbourg (nobody got off the train there)?

Operation in line	Write the answer

Problem b-3: A train leaves Paris. It stops at Le Mans. It starts again for Rennes, its terminus. How many passengers arrive at Rennes?

Write the answer in the blank cell of the table.

Paris, departure	Le Mans, stop	Rennes, terminus
547 passengers get on the train	124 passengers get off the train. Nobody gets on.	
Operation in line:		

Problem c-1: Color the number of passengers in the train at the terminus (in the first column)

Number of passengers in the train				
	800 700 600 500 400 300 200 100 0		200 passengers leave the train. Nobody gets on.	
		Paris, departure	Aix-en-Provence, stop	Marseille, terminus

Time axis

Appendix 2. Pupils' results in CE2 (P3) (207 pupils)

Problem 1 (Pb1)			Problem 2 (Pb2)			Problem 3 (Pb3)		
Txt	Tab	G	Txt	Tab	G	Txt	Tab	G
a-1	b-1	c-1	a-2	b-2	c-2	a-3	b-3	c-3
86	57	25	47	44	21	29	32	20

Significant differences:

- between a-1 (text) and b-1 (table) at the level of 1% (χ^2 = 42), also a-1 and c-1 (graph);
- between a-2 (text) and c-2 (graph), to the advantage of the text, at the level of de 1% (χ^2 = 31);
- between a-3 (text) and c-3 (graph) to the advantage of the text at the level of 5% (χ^2 = 4.72).
- between the problems b (table), better solved than the problems c (graph), at the level of 1% for Pb1 and Pb2 (χ^2 = 15; χ^2 = 9) and at the level of 10% for the Pb3 (χ^2 = 3.16).

We cannot conclude, at the level of 10% (χ^2 = 0.35), to a significant difference between a-2 (text) and b-2 (table), nor between a-3 (text) and b-3 (table) (χ^2 = 0,41).

Appendix 3. Pupils' results in CM1 (P4) (220 pupils)

Problem 1 (Pb1)			Problem 2 (Pb2)			Problem 3 (Pb3)		
Txt	Tab	G	Txt	Tab	G	Txt	Tab	G
a-1	b-1	c-1	a-2	b-2	c-2	a-3	b-3	c-3
89	72	44	49	58	35	52	43	36

The results in Pb2 and Pb3 are not significantly different in the problems a (49%, 52%) or c (35%, 36%).

Significant differences:

- between b-2 (table) and b-3 (table), to the advantage of b-2 at the level of 1% (χ^2 = 10);
- between a-2 (text) and b-2 (table), with an advantage for the table, at the level of 10%, near to 5% (χ^2 = 3.65);
- between a-3 (text) and b-3 (table), with an advantage for the text, at the level of 10%, near to 5% (χ^2 = 3.29);
- between a-2 (text) and c-2 (graph), to the advantage of the text, at the level of 1% (χ^2 = 8.96);

- between a-3 (text) and c-3 (graph), to the advantage of the text, at the level of 2% (χ^2 = 6.40).

Appendix 4. Pupils' results in CM2 (P5) (230 pupils)

Problem 1 (Pb1)			**Problem 2 (Pb2)**			**Problem 3 (Pb3)**		
Txt	Tab	G	Txt	Tab	G	Txt	Tab	G
a-1	b-1	c-1	a-2	b-2	c-2	a-3	b-3	c-3
97	89	74	56	74	65	71	65	70

Significant differences:

- between the texts (χ^2= 9.55 en a), to the advantage of the Pb3, at the level of 1%;
- for the table, to the advantage of the Pb2, at the level of 5% (χ^2= 4.10);
- between a-2 (text) and b-2 (table) to the advantage of the table, at the level of 1% (χ^2 = 16);
- between a-2 (text) and c-2 (graph) to the advantage of the graph, at the level of 5% (χ^2 = 4.01).

Not significant differences:

- between a-3 (text) and b-3 (table) at the level of 10%;
- between a-3 (text) and c-3 (graph) at the level of 5% (χ^2 = 0.04).