Writing Across the Mathematics Curriculum

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In discussing Writing Across the Curriculum, mathematicians and non-mathematicians alike are inclined to ask when and how, if at all, would we use writing in mathematics courses. To begin answering that question, I would say that writing can indeed be incorporated into most of our courses and that we should be making more use of it than we currently do. Furthermore, I would break down the writing in college-level mathematics courses into three categories: ordinary narrative, technical writing, and the writing of proofs. I hope the discussion of these three types will be helpful to mathematics faculty as well as to others who may simply be curious about where writing might fit into the mathematics curriculum. Finally, I have some material of interest to elementary school teachers (and faculty who prepare them) based on observations of mathematics classes in West Germany.

There are two basic themes which emerge in what follows: the use of writing to clarify one's thinking, and the role of writing in helping students express themselves in a precise manner. Both are of concern in all disciplines but are especially crucial in mathematics.

Ordinary Narrative

Let us begin with the mundane considerations. (Warning: English faculty please skip this section. It contains reactionary thoughts about

DOI: 10.37514/WAC-J.1989.1.1.03

stress on mechanics—the stuff that has been boring you to death for years.) In certain courses (e.g. History of Mathematics) we assign papers on topics such as the life and contributions of some highly creative mathematician. These papers call mainly for ordinary narrative as well as some technical writing (discussed below). About ten years ago, my first attempt at assigning such papers did not bring the results that I had hoped for. I naively thought that I would be reading papers that were well thought out, carefully written with proper grammar and correct spelling, documented, typed, and proofread. The results in many cases were none of the above. The attitudes of some students seemed to be that since this was a math course, attention to such matters was not important.

In the intervening years, I have learned that it is necessary to explicitly spell out one's expectations and to reinforce the initial instructions with timely reminders about careful attention to all of the fundamentals mentioned above that make for a paper that is at least readable and possibly even informative. This small amount of extra effort on my part has paid dividends. The papers have been of increasingly better quality. The simple lesson is that in mathematics courses, the forceful laying out of expectations seems to be especially needed to counter that "this-ain't-English" attitude on the part of the students.

Technical Writing

The explanation of some mathematical or statistical procedure or result comes under the heading of technical writing. My students have done a limited amount of this as part of many of the papers described above. Considerable technical writing is also employed in our Applied Statistics course. In that course, students conduct a variety of statistical analyses (hypothesis tests, the fitting of regression equations, etc.) and then report on the results. Such an assignment calls for a written explanation of goals, procedures, and results. This writing, more than ordinary narrative, taxes the writer's abilities to explain technical material in a manner that is precise, yet clear to the reader. This of course is difficult for anyone— not only students—still struggling to fully comprehend all aspects of the material at hand. But then, this leads to one of the most beneficial uses of writing: its use as a thinking clarifier. When confronted with the need to put certain ideas down on paper, one is forced to first clarify those ideas in one's own mind.

And this in turn leads to a very effective use of writing in almost any mathematics course: short verbal explanations asked of students on selected test questions. (Lest this appear too burdensome come correcting time, I hasten to emphasize the word 'selected.') A typical question might read, "If two variables have a correlation coefficient of -0.98, explain the meanings of the negative sign and the absolute value of 0.98." Or we might ask students to verbalize the geometric significance of a gradient vector. Some additional good examples are given in the references. (See King, 1982 and Schillow, 1987.) Telling students in advance that they should expect interpretive test questions will direct their study toward a fuller understanding of concepts as well as computational procedures. It will thus help them realize that critical thinking is more than churning out numbers— that a numerical result is worthless if one is unable to interpret its significance. Reading the responses to these questions also serves the purpose of providing eye-opening feedback on student misconceptions.

The Writing of Proofs

Courses designed mainly for mathematics majors carry a heavy emphasis on proof. Currently at Plymouth State College, the most proof-laden courses are Euclidean Geometry, Non-Euclidean Geometry, Linear Algebra, Algebraic Structures, and Advanced Calculus.

Proofs are far the most difficult writing assignments in mathematics courses. There are several reasons for this, and analyzing those reasons is an instructive exercise in finding ways to help our students become better proof writers. First of all, a good proof can *not* be a rambling discussion, but rather it must be a carefully constructed sequence of logical statements whose end result is the desired conclusion. Each statement must be precise and be a logical consequence of previous statements or other agreed-upon assumptions. This is not easy for anyone who is still struggling to fully understand and sort out all of the interconnections in his or her own mind. And of course most of our students are adolescents who are more accustomed to teenspeak ("It was like totally awesome.")

than to the more demanding task of expressing themselves in a precise manner.

Even when instructed to be precise, and assuming that the reasoning is understood in the student's own mind, there is still a great leap forward required in transcribing those thoughts into a well-written proof. And for this, our students have had very little practice. One obvious reason for the dearth of prior training is the inordinate amount of time needed to teach and correct proofs. But there are other reasons not so readily apparent.

For most of us, our first experience with proofs came in high school geometry. And while we generally had competent teachers, many of our students have been taught by people unqualified for the task due to the chronic shortage of mathematics teachers and the resultant filling of positions with "temporary" help.

Even if a student had good instruction in high school geometry, the format of proofs taught in that course does not usually involve the writing of ordinary English sentences. A typical high school proof might look like the following:

Statement	Reason
p	given
q	Theorem 5
r	Axiom 3
s	SAS

where p,q,r, and s would be statements such as $\angle 1 = \angle 2$.

It is actually a straightforward process to convert such a proof into an English paragraph that does the same job, namely lead the reader from the premises to the conclusion. For the two-column proof above, one equivalent verbal proof would be:

> It is given that p is true and q follows from Theorem 5. Furthermore, Axiom 3 implies statement r. Finally, our conclusion s is a consequence of the Side-Angle-Side Theorem.

I recently tried a modified version of one of Toby Fulwiler's techniques in Algebraic Structures (a course for junior or senior math majors). The particular Fulwiler technique is to have selected student writing samples on transparencies and then project them onto a screen for class discussion. My class was small (twelve students) so I asked the entire class to write out proofs on transparencies (no copying required on my part). As each proof was flashed on the screen, we discussed strengths and flaws. The names were not shown, but of course each student recognized his or her own work so interest was intense. The exercise was enlightening for all concerned, including me. I discovered that the biggest problem was not the write-up, but rather determining what logical steps were needed. They hadn't yet mastered the material underlying the theorem to be proved, and this I believe is an important key to the problem.

We mathematicians often express dismay at the inability of our students to write proofs and frequently imply that proper grounding was not provided in previous courses. Clearly, good prior training is needed, but some elementary material in each new course must be assimilated before one can write a proof incorporating that material. Euclid may have been a master of deductive reasoning but would surely have failed to make sense in a calculus proof before learning a little basic calculus.

Can the above disparate thoughts help us formulate an effective strategy for teaching our mathematics majors to construct well-written proofs? I would suggest that the best approach is to insure that all majors receive instruction on the fundamentals of logic in some early course and then get ample practice with proofs in a variety of contexts throughout the major program. But we must keep in mind that proofs are not created in a vacuum. In order to construct a proof which makes sense, a student must have a clear understanding of the preceding material, and fully comprehend what is to be proved.

Epilogue: Should we start in first grade?

Part of my sabbatical project in 1985 involved visiting mathematics classes in West Germany to determine why German students are so far ahead of their American counterparts. My wife, who is native German, worked with me on this project, thus preventing any possible communication gaps due to my less-than-perfect German. We discovered that the differences were apparent already at the elementary school level. German children are taught more than their American peers in the first grade, and then the gap widens with each successive school year. Multiplication, for example, is a second-grade topic in Germany, whereas American children normally learn their multiplication tables in the third grade. We found that the setting and maintaining of higher expectations has a lot to do with the more rapid progress of German children. However, there are other factors involved, including a variety of teaching techniques.

One of those techniques is the integration of writing and mathematics starting in the first grade. We visited a first-grade class that was getting its initial introduction to fractions. The teacher whetted appetites for what was to follow by bringing to class a chocolate cake which needed to be divided for the birthday of a pair of twins. She used a can of whipped cream to make a line across the middle and then introduced the notion of halving. She then proceeded to explain the following sequence of sentences which she wrote out (in German) on the chalkboard.

Half of ten is five.	Half of six is three.
Half of 10 is 5.	Half of 6 is 3.
1/2 of 10 = 5	1/2 of 6 = 3
$1/2 \bullet 10 = 5$	$1/2 \bullet 6 = 3$

The point I wish to emphasize here is the early age at which a German child learns that a given mathematical equation is equivalent to an ordinary verbal statement. Thinking, writing, and speaking precisely are activities that cut across the curriculum. If they are integrated as in this German first grade math class, a multidimensional stretching of each child's intellectual capacity takes place, and there is no reason why this integrated verbal and mathematical growth cannot also begin in an American first grade. And if this growth can start in the first grade, think of the possibilities for what can be accomplished in the grades that follow.

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