Models in Algebra and Rhetoric: A New Approach to Integrating Writing and Mathematics in a WAC Learning Community

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"We can have words without a world but no world without words or other symbols" (6).
— Nelson Goodman, *Ways of Worldmaking*

This paper documents an ongoing experiment designed to integrate the teaching of college algebra and college rhetoric and writing at Montgomery College in Conroe, Texas. These are the first two college-level math and English courses that students take within the college's core curriculum. Our approach focuses on the concept of models and model building and might be easily adapted to a variety of math and writing classes. We believe we have maintained the necessary rigor of both disciplines while providing a foundation which links them.

The primary aim of this experiment is educational enrichment and enhanced critical thinking. We make no claims that learning to do algebra or to write becomes "easier" as a result of the focus on models. Nor do we claim learning to do math and to write is somehow the same enterprise. Nevertheless, our students have demonstrated that yoking the two disciplines by focusing on models provides a powerful critical instrument they can use to enhance their critical ability across a variety of interdisciplinary contexts.

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Writing as a Full Interdisciplinary Partner versus the Traditional Handmaiden Approach

Traditionally, when a writing course is linked to math or some other technical subject, say biology or engineering, despite the best of intentions, the writing component functions as little more than a handmaiden to the presumed more difficult subject. For example, in a linkage between biology and writing, all too often the writing assignments amount to little more than writing about the biological content. The writing is assumed not be a content course, per se. Students might produce summaries of chapters in the biology book to check their understanding, or write a research paper on a biological topic. In engineering they might learn to write unambiguous accounts of complex problems and solutions. In both instances, however, the writing is merely "at the service" of the other discipline.

This is the case even in as nuanced a presentation as we find in Meier and Rishel's *Writing in the Teaching and Learning of Mathematics*, in which the role of writing is to elucidate the mathematics: "Mathematics is often described by writing faculty as a 'content discipline,' which means, to be blunt about it, there's something there to write about" (3). While acknowledging that "mathematics is embedded in language" (90), Meier and Rishel never extend their view of the "teachable" relation between math and writing to be much more than the need for clear and precise expression. This asymmetry between the disciplines, one the master and the other the handmaiden, becomes the point of departure for our approach, which uses the idea of models to initiate a more interdisciplinary dialogue between the two disciplines.

Learning Community Approach

Our courses are integrated into a block-style "learning community." Such course linkages offer a wonderful opportunity to establish not only disciplinary interconnections but also to bridge traditional curricular and institutional separation. This parallels the approach Zawacki and Williams have called "Writing Across Curricular Cultures" (111) in their recent study of the role of WAC-oriented learning communities. Our community convenes twice a week for 3 hours each meeting. We share the same students and attend the other's "classes." In this way we model by our conduct what the community is about. We use the term "class" loosely because we vary the order of instruction. Some days we do algebra first and some days we begin with the writing. Whichever one goes first, that discussion is often punctuated with observations and questions that connect the one discipline to the "other." Sometimes we proceed more dialogically, with the math and writing more fully intertwined, for example, as we integrate discussion of a reading and some journal writing into an algebra lesson.

The goal is to set out what amounts to a "grammar of algebra" alongside that of a "grammar of rhetoric." More precisely, it is to demonstrate the degree to which the "grammar," that is the traditional manner of thinking and expressing in each discipline, is mirrored in the other. Students come to recognize that algebraic notation is a form of argumentation. It is not just a representational but a persuasive exercise. A particular notation, for example, typically suggests a certain course of action. They also come to realize that learning to write means appreciating the structural relations among writer, audience, topic and context, just to name a few factors.

The special support Montgomery College provides learning communities allows preferential scheduling and a budget for "meet and feed" get-togethers and other social activities (fieldtrips) to enhance the community aspect of learning. The students tend to support one another better in such an environment. If someone is absent a classmate invariably volunteers to email or call him/her to pass on the current assignment. We get to know and value our students much better than in traditional classes.

What is a Model?

We begin with a working definition of model as a "representation of a state of affairs or relations." Such a state of affairs might be mathematical, say the equation $2x+6=25$, economic, historical, literary or rhetorical. To begin, we examine model airplanes, toy soldiers and plastic "models" of all kinds to discover how these conceptually and pragmatically function. The class inductively creates a taxonomy of functions as follows: Models

- represent,
- predict the future,
- imply narrativity,
- persuade,
- reveal, and
- conceal.

The class divides into smaller discussion groups of 3 or 4 to analyze the "show and tell" models we have provided or that the students have brought from home. For example, a model airplane represents the relations between its constituent parts. The model allows us to predict the way an actual plane would look whether or not we have ever seen one. Moreover, the model possesses an implicit narrative component; that is, we cannot help but remember or perhaps imagine some experience of a plane, or of something familiar that will help us interpret the model. This could be based on personal experience or just imagination. The point is that some reliance on narrative is implicit in both the structure of the model as well as in our process of understanding it. The model also includes a very powerful persuasive aspect in that we thus come to believe, on the basis of the model, what an airplane is. We remain persuaded until another purported model of the same entity appears on the scene and challenges our assumptions.

The two aspects of models that we focus on the most are that any model necessarily reveals at the same time that it conceals. Models are always provisional. Like stories and other verbal constructs, or even history, they are constructed from a point of view. For example, the model airplane may not include any of the internal engineering needed for it to fly. Thomas Kuhn's notion of the conceptual "paradigm" or "accepted model or pattern" (23) comes to mind; that is, a set of assumptions that serve as conditions of possibility for what we take to be "real." The history of science, as well as of algebra and rhetoric, is essentially the history of competing paradigms, each built and advanced from alternative points of view. For example, the gradual acceptance of imaginary numbers in algebra is one such instance of this.

Early in the course when we introduce models and model building, we read Henry Louis Gates' essay, "What's in a Name?" (5-6). This autobiographical piece about the power of names in the construction of (personal) identity allows us to move from visual to verbal models. Knowing the name of someone or something influences our understanding of and interaction with the person or thing. The act of naming is itself a model building activity. What's in a name? As Shakespeare might observe, nothing, and everything.

Knowing or recognizing the name of an algebraic equation, function or relation sets in motion "orders of operations" necessary to solve for an unknown or to represent the mathematics in alternative ways, say in a linear as opposed to a graphic model. That is, learning algebra is largely becoming aware of the different kinds of algebraic and other mathematical models that allow us to "do" algebra. Not only this, it allows us to understand that how we solve and represent a problem and its solution is itself to construct a model, especially in that many problems may be solved in different ways. Doing algebra is using and building models no less than is learning to write.

Models and Rhetoric: "Modeling to Learn"

The writing and rhetoric components of the learning community are deeply embedded in the language of modeling. For instance, at the beginning of the course when presenting "strategies of invention" or ways to get started, we portray freewriting, brainstorming, clustering and outlining as model building activities. These are ways to represent thinking. To outline is to construct a model, say of an essay or an argument. The very idea of an essay is to "assay" or "test" a set of ideas. What is a draft but a model under construction? Close reading, which we stress in both the rhetoric and algebra components, means to build a virtual model of textual interpretation. Students become intrigued, as a matter of fact, to encounter our insistence on reading ordinary "word problems" as closely as if they were poems. The fear many have of this special kind of math problem is in fact often due to not knowing how to read carefully. We encourage them to draw pictures and model what they find in the language before rushing to solve the problem.

Aristotle's familiar topics or modes of rhetorical presentation are not only ways to model forms of expression but also the mind in the process of engaging the world. The familiar strategies of narrative, cause and effect, definition, classification and division, comparison and contrast, as well as deduction and induction are all ways we build models and thus create coherence out of the flow of our experience. Each strategy manifests its own power and limitation to represent; it both reveals and conceals at the same time.

The units of the rhetoric and writing part of the course are as follows:

- Models of self, family and community,
- Models of play, literature and art,
- Models across the curriculum,
- Models of language and gender,
- Models in media,
- Models in politics and the postmodern world.

One of the first writing assignments is for each student to construct a model of his/her family, embedding or positioning themselves within this structure. They may use any graphic means they wish to accomplish this although most create a standard genealogical tree model. Some prefer to use Venn diagrams. They are then asked to describe the powers and limitations of this model to represent the "real" situation at home: Who actually has the most power and influence? Does the model reflect any outside factors such as grandparents or divorced parents? Needless to say, in today's complex social world, these models can get quite involved. There are typically lines of filiation going every which way in these models.

Next, the students ask a member of their immediate household to draw a family model from their point of view so they can compare it with the earlier one. It is always enlightening to see the "same" reality modeled in an alternative manner. Finally, we ask the students to translate, as best they can, their original model into an algebraic expression such as the following: X+[(Y+1/2D)+(1/2S+1/3G)]=1

Let X be a divorced parent living outside the family unit while the other parent, Y, raises the three children, D, S and G, each with varying degrees of connection to this parent. The point is to explore both the advantages and disadvantages of translating one way of modeling, graphic, into another, linear. Typically students discover that by making a model they actually come to an enhanced understanding of their particular family dynamic. That is, they gain insight they could not have achieved without the advantage of the cognitive distancing effect the model affords: "I never realized how much our two relationships affected the entire family, positively and negatively" (used by permission).

In the unit on play we take Johan Huizinga's discussion of this concept in *Homo Ludens* (1-27) and use it to write analyses of any number of "rule-governed" activities, such as going to war, advancing a lawsuit, making music, or even shopping. We want the students to learn how to apply a "totalizing" model of human behavior to represent a variety of contrary activities. In so doing, the students expose both the explanatory power and severe limitation of Huizinga's model, which they discover manifests great utility and yet in the final analysis is just one provisional model of culture and human action among many others.

Our concern with play opens up to a unit on learning to read literary texts, especially drama and poetry. We present literary genres, say epic, tragedy, comedy, or lyric, as alternative ways to model human experience that a writer selects to suit his/her personal as well as social purposes. Such choices are not unlike the ones any writer makes to build a text. Interpreting literature includes hermeneutically learning to “play by the rules” of a particular genre, for instance the sonnet, while not completely limiting oneself to those “rules.”

In our unit on media and advertising the students learn to deconstruct the consumerism of ads and commercials by considering the models of gender, age, sex, and social class that advertisers use. They write papers in which they explore how alternative images model personal and social forms of desire.

Following our unit on models across the curriculum, which we explain in a separate part of this paper, we conclude the semester’s reading and writing with two ways to model postmodern experience. We focus on two key models, that of “entropy” in Thomas Pynchon’s short story, “Entropy,” and Don De Lillo’s trope of “white noise” in his novel, *White Noise*. Students explore the power and limitation of these alternative “lenses” through which they scrutinize the contemporary political and cultural scene.

Models in Algebra: “Learning to Model”

The following examples from algebra illustrate selected Aristotelian modes or topics. We stress to the students the degree to which these conceptual and rhetorical strategies model thinking in both disciplines. We use algebra to model Aristotle’s modes as an introduction to the nature of algebraic argumentation and persuasion. In this way the algebra ironically seems “to serve” the rhetoric.

Narrative

How do some topics in algebra illustrate narrativity? We emphasize that reading some algebraic expressions is like reading a narrative in that we establish an “order of operations,” a computational order, and the relationships between the parts in the expression. This allows us to demonstrate the coherence of the form as a whole. As in literature, the ability to recognize the genre of expression aids the reader in being able to interpret the text.

For example, in algebra the formalism $(f \circ g)(x) = f(g(x))$ stands for the composition of two functions f and g. The genre here is what is called “the algebra of functions.” The formula is intended to be read as a sequence of events, as in a narrative, namely, first compute the number $g(x)$ and then use this number to compute $f(g(x))$.

This step-by-step process is similar to an auto assembly line: first put on the wheels, then the engine, then the doors, etc.

Another example would be how to interpret $f(x) = (x-3)^2 + 1$, to sketch the graph of this function. This notation is written to suggest that the student first draw a basic parabolic shape, move this shape to the right 3 units, and then lift it up 1 unit. The sequence of graphs tells the "story" (See Figures 1-3 below).

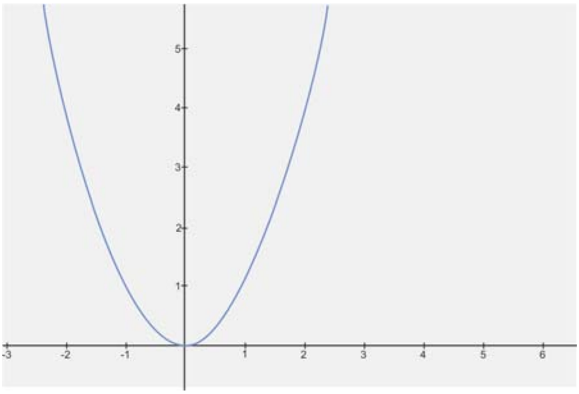

Figure (1) $y = x^2$

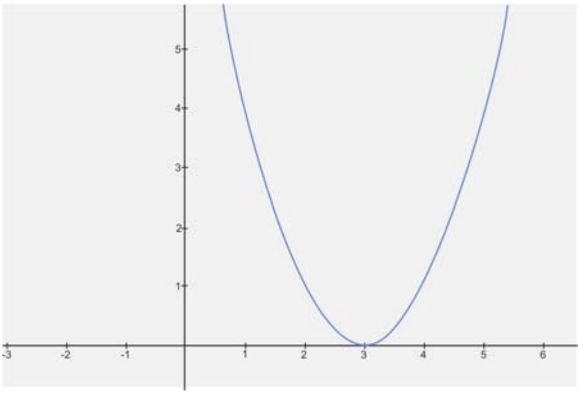

Figure (2) $y = (x-3)^2$

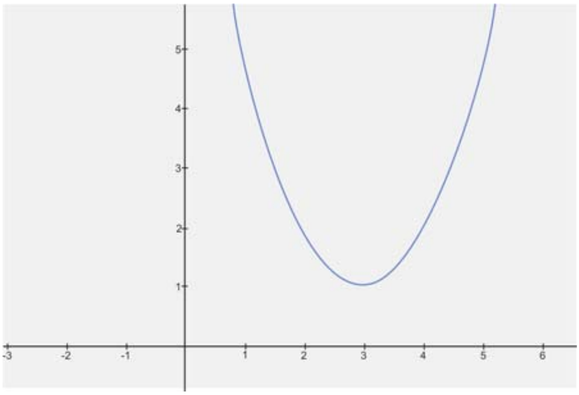

Figure (3) $y = (x - 3)^2 + 1$

Cause and Effect

What is an example in algebra that illustrates cause and effect? One example would be the problem of finding the intercepts of the graph of an equation. The idea is this: an equation in two variables determines some curve. The problem is to find the coordinates of the points where this curve crosses the horizontal (x-axis) and the vertical (y-axis). To find where the curve crosses the x-axis, the student sets the y variables in the equation to zero. This cause then has the effect of allowing the student to solve the equation for x. This number (or numbers) is (are) where the curve crosses the x-axis. The student uses a similar procedure to find where the curve crosses the y-axis

Definition

Being able to do algebra effectively requires a firm grasp of definition and the notation used to represent definitions. Before a student can evaluate any expression or solve any equation he/she must be able to recognize the assumptions implicit in the notational set-ups.

For example, consider the equation $2^x = 7$. To solve this for x, the student must use two definitions. First, the student uses the definition of logarithm base 2. Basically, the logarithm base 2 of a number, $x = \log_2(7)$, is an exponent so that 2 raised to this exponent is 7. In other words $x = \log_2(7)$ is defined to mean $2^x = 7$. The student must then use a second definition to rewrite logarithm base 2 into a quantity that can be put into a calculator to get the final answer. The second definition is

$$x = \log_2(7) = \frac{\log(7)}{\log(2)}$$

or approximately 2.8073.

Classification and Division

As an example of classification and division, the student learns to recognize differences between equations and group them in types, for example, linear equations, quadratic equations, quadratics in form, and exponential equations. Knowing these types aids the student in selecting appropriate techniques to solve the equations. Each of the equations listed below is an example of the four types listed above.

- $3(x-2)+6=5x+2$
- $x^2-5x+1=0$
- $x^4-5x^2+1=0$
- $2^x=7$

Comparison and Contrast

The student uses this mode in the example of the so called "symmetry tests" for the graph of an equation in two variables x and y. For example, a curve is said to be symmetric with respect to the vertical (y-axis) if the y-axis acts like a mirror between two halves of the curve. To use the test, the student compares and contrasts an equation in x and y with a new equation obtained by replacing x by –x. If the two equations are equivalent, the graph of the original equation will be symmetric with respect to the y-axis.

Deductive Reasoning

Deductive reasoning moves from the general to the specific. An algebra student would use deductive reasoning when applying, for example, the quadratic formula. This formula essentially states that the solution to any quadratic equation in the form $ax^2+bx+c=0$ is given by the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$. For example, the student would use this formula to solve, say, $3x^2 + 5x + 1 = 0$, by first identifying a = 3, b = 5, and c = 1, and then substituting these into the formula.

Inductive Reasoning

Going the other way, from specific to general, an algebra student would use this reasoning in describing all the solutions to the linear inequality problem $2x + 1 \leq 11$. In this problem, the student is asked to describe all numbers x so that 2 times x plus 1 is less than or equal to 11. The student could verify that this works for x = 1, 2, 3, 4, 5 but not for x = 6, 7, … By studying these individual examples, the student can then reason that the solution is all numbers less than or equal to 5.

Multiple Representations

A running theme of our community is to emphasize how we use models to make multiple representations of a state of affairs. Here is an example of this from algebra.

Problem: Find the domain of the function $f(x) = \frac{1}{x-2}$. (The domain of a function is the set of all numbers x so that the right-hand side of the function is a defined number. In this case, since a fraction cannot have zero in the denominator, x cannot be equal to 2.)

The answer may be written in many different ways; four of them are as follows:

(1) All real $x \neq 2$

(2) $\{x \in \Re \mid x \neq 2\}$

(3) $(-\infty, 2) \cup (-2, \infty)$

(4) 

We emphasized in class discussion the advantages and disadvantages of each of these models. For example, the sentence (1) is easy to read. The set notation (2) emphasizes the fact that the domain of a function is a set of numbers. The interval notation (3) provides a notion of directionality and scale on the number line, and the graph (4) gives a visual sense to the solution not readily available in the other three models.

Models Across the Curriculum

Near the middle of our semester we invite several guest speakers to the class to present the use of models in their disciplines. Each provides a brief reading for the students and moderates a discussion. Our goal is for the students to see first hand the way professionals in a variety of fields use models in their daily work.

Dr. Betsy Powers, a social historian, discussed the idea of history itself as always linked to competing models of time, say linear, cyclic, upward or downward trending. She also brought in some demographic data and guided the students as they profiled the population of a small mid-western farming community and village based on these census “facts.”

Dr. Olin Joynton, a philosopher, discussed Descartes’ preference for an architectural metaphor of building or laying a solid foundation in the latter’s search for epistemological certainty.

Dr. Sunita Cooke, a biologist, led a discussion of the modern scientific or experimental method as a model of exploration. She then discussed the way biologists and other scientists construct and test models as part of their experimental work.

Finally, Mark Stelter, a professor of criminal law and an attorney, discussed the ways judges rely on particular models of interpretation in order to “make sense” of the Constitution.

Fieldtrip

We visit the Houston Museum of Science to explore how the various models of scientific principles displayed there help us “see” what otherwise would be invisible and thus harder to comprehend. For example, we spend considerable time at Foucault ‘s Pendulum, a model that allows us to experience what we cannot readily observe, namely, the rotation of the earth under our feet. We ask the students to do some calculations using the pendulum and they also write about how various scientific models on display both reveal and conceal at the same time. For example, the model of the solar system we view on the front steps and portico of the museum, stretching a hundred feet or so in length, adequately represents the relative distances between the planets but conceals the vast spaces which are really between each planet. The model necessarily displays and distorts at the same time.

While at the museum, typically at the chemistry exhibit, we also consider the aesthetic implications of certain scientific models, for example the display of different chemical bonds and structures. We ask the students to reflect on what would lead them to consider a scientific model beautiful, if not a work of art. This opens up to considerations about the nature of form, elegance and beauty which allows us to discuss the aesthetic implications of scientific model building.

Group Research Projects

During the final three weeks or so we divide the community into research teams of 3 or 4 students. Each team explores the application of a particular mathematical model in a discipline, for example physics, chemistry, medicine or even political science.

The students read about the model, study all the variables in the algebraic notation for the model including units of measure, and work some numerical examples of the formula. They also make graphs of the model where applicable. Physical phenomena are in general functions of many variables. The main value of a mathematical model is that it helps the students see which variables are most significant. This helps them understand what the model reveals and conceals.

Each group divides the labor and writes up what they have done, focusing especially on the alternative ways they discover they have to display their solutions. They make an oral presentation to the community using PowerPoint. Some of the projects to date are as follows:

- Model of Richter Scales,
- Model of Decibel Scale,
- Model of Radioactive Decay,
- Drug Metabolism,
- Newton's Law of Cooling,
- Doppler Effect (sound waves),

- Bernoulli's Equation (fluid pressure),
- Model of Weighted Voting Systems, and
- Model of How to Generate Drivers' License Codes.

The WAC Initiative at Montgomery College sponsors a Student Presentation and Critical Thinking Conference each semester, and one or two of our teams invariably present their work at this meeting, to which the entire college community is invited.

Learning Outcomes

In her cover letter to her final course portfolio, one of our students writes as follows: "Our study of models has . . . made me look deeper into things than I would have before. . . . Models are what we look to for guidance and inspiration, and now I have more insight as to how everything really does relate to each other" (used by permission).

By the end of the semester, as evidenced in their group projects and in their final portfolio statements, the majority of the students "got it." There were no sudden breakthroughs. None were expected. However, we did see a gradual and palpable consciousness raising over the course of the semester. Did they learn to do algebra and improve their writing? Yes. All the curricular outcomes for college algebra and writing were met. But it is what else they learned that matters most.

The key outcome we discovered in our students was their increased awareness of their daily reliance on models of all kinds. Even more important, they came to see themselves not as passive users but active builders of models. We saw this most significantly, of course, in the way they began to think and express themselves about algebra and rhetoric. They possessed a vocabulary that allowed them to solve problems in both disciplines and then discuss the power and limitations of the way they chose to solve them.

They came to appreciate, moreover, the value of trying to understand one thing in terms of another, which is the very nature of metaphor and modeling. In so doing they became at once more acute critics of models as historical and provisional constructions and therefore more comfortable with the idea of ambiguity and multiple interpretations.

The key outcome was their demonstration, especially in their final group projects, of the multiple ways they could model and meaningfully assess their variable solutions to the discipline-specific algebra problems. For example, each group was charged to discuss the strengths and weaknesses of presenting their solutions in different ways, say in a graphic or linear manner. This was as important as correctly solving the problem. They also became aware that the format they used to write up their final projects was itself a model, a conventional form for reporting experimental results.

Although they obviously wrote "about" algebra we avoided having the writing merely serve the mathematics because the real object of our linked courses was model building, and not merely algebra or rhetoric/writing. The focus on models enabled us to elude the either/or situation that traditionally separates mathematics and the humanities, and enabled us to make connections where traditionally few if any have been presumed to exist.

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